[B] relates / couples the force and moment terms to strains and curvatures.

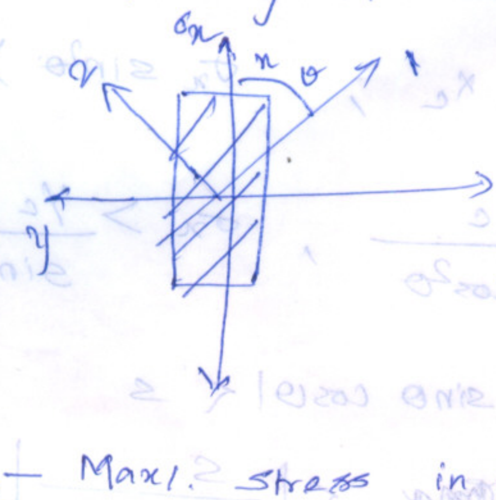
Failure theories:

* Maximum stress failure criterion
* Max. strain failure criterion
* Tsai - Hill failure criterion
* Hoffman failure criterion
* Tsai - Wu tensor failure criterion

(i) Max. σ failure criterion:

The stresses in the principal Material co-ordinates must be less than the respective strengths, Otherwise fracture will occur.

(i) If σ_x alone in applied, then
$\sigma_y = \tau_{xy} = 0$.



Take, $X_{t/c}$ – Max. stress in tension or comp. in

S – Maxi. Shear stress in ~~direction~~ 1-2 plane.

$Y_{t/c}$ – Maxi. stress in tension / comp in direction 2.

According to the theory,

$$\sigma_1 < X_t \quad , \quad \sigma_2 < Y_t$$

$$\sigma_1 > X_c \quad , \quad \sigma_2 > Y_c \, .$$

$$|\tau_{12}| < S$$

We know,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -sc & sc & c^2-s^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad \Bigg\| \begin{matrix} \sigma_y = 0 \\ \tau_{xy} = 0 \end{matrix}$$

$$\Rightarrow \sigma_1 = \sigma_x \cos^2\theta \, , \quad \sigma_2 = \sigma_x \sin^2\theta$$

$$\tau_{12} = -\sin\theta \cos\theta \, \cancel{\tau_{xy}} \, \sigma_x$$

$$\therefore \sigma_x = \frac{\sigma_1}{\cos^2\theta} \, , \quad \sigma_x = \frac{\sigma_2}{\sin^2\theta} \, , \quad \sigma_x = \frac{-\tau_{12}}{\sin\theta \cos\theta} \, .$$

also,

$$\sigma_x \cos^2\theta < X_t \, , \quad \sigma_x \sin^2\theta < Y_t \, .$$

$$\sigma_x < \frac{X_t}{\cos^2\theta} \, , \quad \sigma_x < \frac{Y_t}{\sin^2\theta} \, .$$

$$\sigma_x \cos^2\theta > X_c \, , \quad \sigma_x \sin^2\theta > Y_c$$

$$\sigma_x > \frac{X_c}{\cos^2\theta} \, , \quad \sigma_x > \frac{Y_c}{\sin^2\theta} \, .$$

$$|\sigma_x \, \cancel{\tau_{xy}} \, \sin\theta \cos\theta| < S$$

$$|\sigma_x| \, \cancel{\tau_{xy}} < \left| \frac{S}{\quad} \right|$$

∴ The applied stress σ_x should be,

$$\frac{X_c}{\cos^2\theta} < \sigma_x < \frac{X_t}{\cos^2\theta}.$$

$$\frac{Y_c}{\sin^2\theta} < \sigma_x < \frac{Y_t}{\cos^2\theta}$$

$$|\sigma_x| < \left| \frac{S}{\sin\theta \cos\theta} \right|$$

to avoid fracture.

(ii) Maxl. strain ~~to~~ Failure criterion:-

The Material is said to be failed if one or more of the following inequalities is not satisfied.

$$\epsilon_1 < X_{\epsilon t} \qquad \epsilon_2 < Y_{\epsilon t}$$

$$\epsilon_1 > X_{\epsilon c} \qquad \epsilon_2 > Y_{\epsilon c}$$

$$|\gamma_{12}| < S_\epsilon$$

$X_{\epsilon t}/_{\epsilon c}$ — Maxl. strain in tension / compression in direction 1.

$Y_{\epsilon t}/_{\epsilon c}$ — Maxl. strain in tension / compression in direction 2.

S_ϵ — Maxl. shear strain in 1-2 co-ordinates

(8)

When unidirectionally reinforced composite material is subjected to uniaxial load at an angle θ to the fiber orientation, then the allowable stresses can be calculated from allowable strain,

$$\epsilon_1 = \frac{1}{E_1}(\sigma_1 - \nu_{12}\sigma_2)$$

$$\epsilon_2 = \frac{1}{E_2}(\sigma_2 - \nu_{21}\sigma_1)$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

We know,

$$\sigma_1 = \sigma_x \cos^2\theta, \quad \sigma_2 = \sigma_x \sin^2\theta, \quad \tau_{12} = -\sigma_x \sin\theta\cos\theta$$

$$\epsilon_1 = \frac{1}{E_1}(\cos^2\theta - \nu_{12}\sin^2\theta)\sigma_x$$

$$\epsilon_2 = \frac{1}{E_2}(\sin^2\theta - \nu_{21}\cos^2\theta)\sigma_x$$

$$\gamma_{12} = -\frac{\sigma_x \sin\theta\cos\theta}{G_{12}}$$

and,

$$X_t = E_1 X_{\epsilon t} \Rightarrow X_{\epsilon t} = \frac{X_t}{E_1}$$

III^{ly},

$$Y_{\epsilon t} = \frac{Y_t}{E_2}, \quad X_{\epsilon c} = \frac{X_c}{E_1}, \quad Y_{\epsilon c} = \frac{Y_c}{E_2}$$

$$S_\epsilon = \frac{S}{\ }$$

Max. strain failure criterion is,

$$\frac{1}{E_1}\left(\cos^2\theta - \nu_{12}\sin^2\theta\right)\sigma_x < \frac{X_t}{E_1}$$

ie, $$\sigma_x < \frac{X_t}{\cos^2\theta - \nu_{12}\sin^2\theta}$$

Illy

$$\sigma_x < \frac{Y_t}{\sin^2\theta - \nu_{21}\cos^2\theta}$$

$$\sigma_x > \frac{X_c}{\cos^2\theta - \nu_{12}\sin^2\theta}$$

$$\sigma_x > \frac{Y_c}{\sin^2\theta - \nu_{12}\cos^2\theta}.$$

$$|\sigma_x| < \left|\frac{S}{\sin\theta\cos\theta}\right|$$

ie,

$$\frac{X_c}{\cos^2\theta - \nu_{12}\sin^2\theta} < \sigma_x < \frac{X_t}{\cos^2\theta - \nu_{12}\sin^2\theta}$$

$$\frac{Y_c}{\cos^2\theta - \nu_{12}\sin^2\theta} < \sigma_x < \frac{Y_t}{\cos^2\theta - \nu_{12}\sin^2\theta}$$

$$|\sigma_x| < \left|\frac{S}{\sin\theta\cos\theta}\right|$$

The difference b/w Max/. Stress and Max/. Strain failure criterion is the the inclusion of poisson's ratio in strain criterion.

(iii) Tsai-Hill failure criterion :

Hill proposed a yield criterion for orthotropic Materials

$$(G+H)\sigma_1^2 + (F+H)\sigma_2^2 + (F+G)\sigma_3^2 - 2H\sigma_1\sigma_2 - 2G\sigma_1\sigma_3 - 2F\sigma_2\sigma_3 + 2L\tau_{23}^2 + 2M\tau_{13}^2 + 2N\tau_{12}^2 = 1 \quad \text{— (A)}$$

This yield criterion stands for the orthotropic strength or the failure criterion based on the spirit of the limits.

The yield stresses F, G, H, L, M, N will be regarded as failure strengths, and they are related to usual strengths X, Y and S for a lamina by T-sai.

(i) If only τ_{12} acts, then

$$2N = \frac{1}{S^2}, \quad 2L = \frac{1}{R^2}, \quad 2M = \frac{1}{T^2} \quad \text{— (1)}$$

(ii) If only σ_1 acts, then $G + H = \frac{1}{X^2}$ — (2)

only σ_2 acts, then $F + H = \frac{1}{Y^2}$ — (3)

(2) + (3) $\Rightarrow 2H + G + F = \frac{1}{X^2} + \frac{1}{Y^2}$

$$2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$$

111ly,

$$2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}$$

$$2G = \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}$$

(B)

Sub (B) in (A)

$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\sigma_3^2}{Z^2} - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma_1\sigma_2$$

$$-\left(\frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}\right)\sigma_1\sigma_3 - \left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right)\sigma_2\sigma_3 +$$

$$\frac{1}{R^2}\tau_{23}^2 + \frac{1}{T^2}\tau_{13}^2 + \frac{1}{S^2}\tau_{12}^2 = 1.$$

For the plane stress in plane 1-2,

$\sigma_3 = \tau_{13} = \tau_{23} = 0$ and

for transverse isotropy $Y = Z$

$$\therefore \frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1.$$

This is the governing failure criterion theory of Tsai-Hill.

For off axis cntd. :-

We know, $\sigma_1 = \sigma_x \cos^2\theta = \sigma_x c^2$

$$\sigma_2 = \sigma_x \sin^2\theta = \sigma_x s^2$$

$$\tau_{12} = -\sigma_x \sin\theta \cos\theta = -\sigma_x cs$$

Then T-Sai ~~Hill~~ theory is,

$$\frac{c^4}{X^2} + \left[\frac{1}{S^2} - \frac{1}{X^2}\right] c^2 s^2 + \frac{s^4}{Y^2} = \frac{1}{\sigma_x^2}.$$

If $<$ is safe

$=$ is not safe.

— More applicable for the failure prediction of E-glass epoxy CMs.

Hoffman failure ~~f~~ criterion:-

$$C_1(\sigma_2-\sigma_3)^2 + C_2(\sigma_3-\sigma_1)^2 + C_3(\sigma_1-\sigma_2)^2 + C_4\sigma_1 +$$

$$C_5\sigma_2 + C_6\sigma_3 + C_7\tau_{23}^2 + C_8\tau_{31}^2 + C_9\tau_{12}^2 = 1$$

Here the 9 constants $C_1, \ldots C_9$ are determined from 9 strengths in principal Material co-ordinates $X_t, X_c, Y_t, Y_c, Z_t, Z_c, S_{23}, S_{31}, S_{12}$.

For plane stress cndn.:

$$\sigma_3 = \tau_{13} = \tau_{23} = 0$$

for transverse isotropy,

$$Y_t = Z_t, \quad Y_c = Z_c, \quad S_{31} = S_{12}$$

The failure criterion reduced to,

$$\frac{-\sigma_1^2}{\;} + \frac{\sigma_1\sigma_2}{\;} - \frac{\sigma_2^2}{\;} + \frac{X_c + X_t}{\;}\sigma_1 + \frac{Y_c+Y_t}{Y_c Y_t}\sigma_2$$

505 (5)

3.

T-sai - Wu failure criterion:

In this process new strength definition is required to represent the interaction between stresses in two directions.

failure surface in six dimensional stress space exists in the form,

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad , \quad i, j = 1, 2 \cdots 6$$

↓

Very complicated.

∴ Under plane stress condition, the equation of an orthotropic lamina is,

$$\underbrace{F_1 \sigma_1 + F_2 \sigma_2 + F_6 \sigma_6}_{\text{Represents different strengths in tension and compression.}} + \underbrace{F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \sigma_6^2}_{\text{Represents ellipsoid in stress space.}} + \underbrace{2F_{12} \sigma_1 \sigma_2}_{\text{Interaction between normal stresses in 1 and 2.}} = 1$$

Under tensile load:

engineering strength = X_t

Under compressive load:

engineering strength = X_c

eqn under tensile load,

$$F_1 X_t + F_{11} X_t^2 = 1 \quad \text{—} \; (1)$$

Under compressive load,

$$F_1 X_c + F_{11} X_c^2 = 1 . \quad — (2)$$

Upon simultaneous solution of eqns. (1) & (2),

$$F_1 = \frac{1}{X_t} + \frac{1}{X_c} , \quad F_{11} = -\frac{1}{X_t X_c}$$

"ly

$$F_2 = \frac{1}{Y_t} + \frac{1}{Y_c} , \quad F_{22} = \frac{-1}{Y_t Y_c}$$

"ly

$$F_6 = 0 \quad \text{and} \quad F_{66} = \frac{1}{S^2} \quad (\text{Shear strength}).$$

For equal strength in tension and compression,

$$X_t = -X_c \quad ; \quad Y_t = -Y_c$$

$$\therefore \quad F_1 = 0 , \quad F_{11} = \frac{1}{X^2} , \quad F_2 = 0 , \quad F_{22} = \frac{1}{Y^2} .$$

Thus the failure criterion is,

$$\frac{\sigma_1^2}{X^2} + 2 F_{12} \sigma_1 \sigma_2 + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 .$$

T-sai wu criterion characteristics :.

(1) Increased curve fitting capability over the T-sai Hill and Hoffman criteria.

(2) Additional term F_{12} can be determined only with an expensive biaxial test.

(3) Graphical interpretations of the results are facilitated by tensor formulation.